Life Sciences Outreach Faculty Speaker Series for High School Biology Teachers
How Biologists View Structure and Function
Fall 2018

Isometry exercise reference sheet

Logarithms and slope magic

We just learned how volume and area change with relation to length under conditions of **isometry** (that is, size is all increasing proportionally, with no changes in shape).
Our cubes behave according to **power laws.** For example, area is proportional to length^2 (e.g., cm^2) while volume is proportional to length^3 and SMoA is proportional to length^4. As we've seen, this leads to a curving line when plotted.
If we take the **logarithm** of these values and plot them, they make straight lines! This is because of how logarithms work. If V is volume, L is length, and b is a constant we don't particularly care about, then:

$V = b*L^3$ is the same as $\log(V) = \log(b) + 3\log(L)$

The equation on the left describes what you made when you plotted volume (y-axis) versus length (x-axis). It's a curving line, as your results show.
The equation on the right describes a plot of log volume (y-axis) versus log length (x-axis), aka a log-log plot. This turns out to be a straight line of slope 3, because the power of volume is 3 (e.g., cm^3). The equation on the right is also just the equation for a straight line ($y = mx + b$).

Answer question 8 in packet B____

Why does this help us? If we know the units of the things we're plotting, then we can know the expected **slope** of a log-log plot under **conditions of isometry**. It's simply the power of the y-axis divided by the power of the x-axis.
So, a log-log plot of how length (y) changes with volume (x) has slope 1/3. A log-log plot of the inverse, how volume (y) changes with length (x), has slope 3/1.

Answer question 9 in packet B____

In biology, volume is proportional to the **mass** of an animal (so we can swap out mass for volume in these equations). Furthermore, **area** applies to more than just surface area; for Brianna's study, she focused on the **cross-sectional area** (of bones), which scales the same and is more relevant to biomechanics.

When we say a "bigger" horse, we generally mean one with more mass. So in the following question, we're going to focus on cross sectional area and second moment of area - but instead of how they scale with length like we just did in the cubes, we'll look at how they scale with mass. (Length does have a role in the forces on bones, but we're not going to worry about that for now.)

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A Nature resource on other biological examples of allometry:
https://www.nature.com/scitable/knowledge/library/allometry-the-study-of-biological-scaling-13228439

Link to Brianna's paper:
http://rspb.royalsocietypublishing.org/content/284/1861/20171174

Link to all of Brianna's data (it's all available!):
http://dx.doi.org/10.5061/dryad.4v130

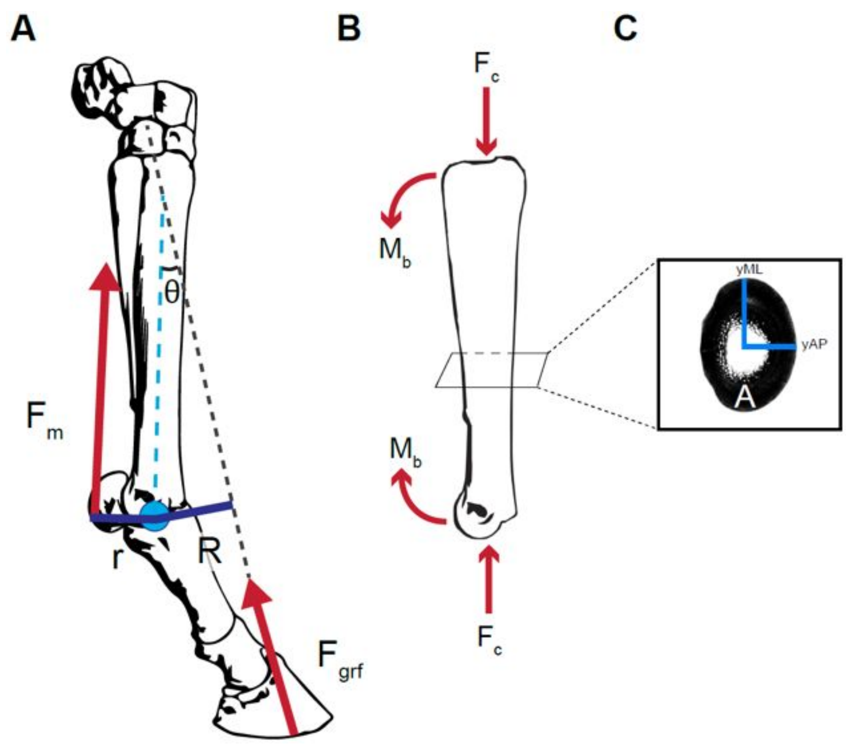

Figure 1. This figure shows how the forces exerted by a horse as it's moving (as shown by F_{grf}) interact with the bones of the lower leg. See how that force is at an angle (in part A)? That translates to some compressive forces (F_c in part B) and some bending forces, which are also called moments (M_b in part B). The **cross-sectional geometry** of that bone (shown as a slice in C) is what determines how well the horse's bone can resist those forces.